

# Creep resistance of the directionally solidified ceramic eutectic of $\text{Al}_2\text{O}_3/\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$ : experiments and models

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## Abstract

The creep resistance of the directionally solidified ceramic eutectic of  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  was studied in the temperature range of 1200–1520 °C both experimentally and by mechanistic dislocation models. The topologically continuous majority phase of  $\text{Al}_2\text{O}_3$ , has a nearly perfect growth texture in the [000 1] direction and encapsulates the minority  $\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  phase in a variety of morphologies. This encapsulated minority phase too has a close to  $\langle 112 \rangle$  growth texture, regardless of morphology. The two phases are separated by close to coherent and well structured interfaces.

The creep of the eutectic in its growth direction exhibits an initial transient that is attributed to stress relaxation in the  $\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  phase, but otherwise in steady state shows many of the same characteristics of creep in sapphire single crystals with  $c$ -axis orientation. The creep strain rate of the eutectic has stress exponents in the range of 4.5–5.0 and a temperature dependence suggesting a rate mechanism governed by oxygen ion diffusion in the  $\text{Al}_2\text{O}_3$ . While required TEM evidence is still incomplete, finite element analysis of stress distribution in the two phases and a detailed dislocation model of the creep rate indicate that much of the nano-scale encapsulated  $\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  is too small to deform by creep so that the major contribution to the recorded creep strain is derived from the diffusion-controlled climb of pyramidal edge dislocations in the  $\text{Al}_2\text{O}_3$  phase. The evidence suggests that the climbing dislocations in  $\text{Al}_2\text{O}_3$  must repeatedly circumvent the  $\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  domains acting as dispersoids resulting in the stress exponents larger than 3. The creep model is in very good agreement with the experiments. © 2005 Elsevier Ltd. All rights reserved.

*Keyword:*  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2$  eutectics

## 1. Introduction

As Fleischer<sup>1</sup> noted, to reach beyond the capabilities of the present set of high temperature superalloys and some intermetallic compounds that have service limitations at around 1000 °C, it is necessary to consider other compounds such as oxides, carbides, borides, etc. to reach service temperatures in the 1400–1600 °C range. While these are all intrinsically brittle materials, having low temperature brittleness problems, their high temperature performance is largely governed by their creep resistance and fracture resistance. It is creep resistance that is of principal concern here to us.

For some time single crystal sapphire fiber with [000 1] axis orientation has been considered as an ideal material for high temperature application.<sup>2</sup> In that orientation with its principal basal and prismatic systems unstressed, sapphire single crystal fibers have remarkable creep resistance in the 1400–1600 °C temperature range where only the pyramidal slip system is stressed. Early experiments of Firestone and Heuer<sup>3</sup> on [000 1] axis-oriented-sapphire in the 1600–1800 °C range produced evidence that such sapphire crystals creep entirely by the climb of the  $1/3 \langle \bar{1}101 \rangle$  pyramidal edge dislocations, with no slip line or stereo-TEM evidence of glide of such dislocations on any of the possible pyramidal planes available. That such dislocations are entirely sessile in glide has now been established in very recent MD simulations of the core structures of these dislocations.<sup>4,5</sup> The stress dependence of the creep rates of [000 1] oriented sapphire crystals

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and their governing activation energy of oxygen ion diffusion in  $\text{Al}_2\text{O}_3$  have all been consistent with a pure climb mode of creep.<sup>3</sup> This is extremely rare in the creep of metals, with the only similar occurrence having been reported by Edelin and Pourier<sup>6</sup> in similarly oriented Mg crystals.

While oriented sapphire single crystal fibers have advantageous characteristics and have been considered with diffusion barrier coatings as reinforcing elements in polycrystalline  $\text{Al}_2\text{O}_3$  matrixes for composite applications,<sup>7,8</sup> the practice has been costly and lacked flexibility. As an alternative to sapphire-fiber-reinforced ceramic composites, a series of directionally solidified ceramic eutectics, consisting largely of an  $\text{Al}_2\text{O}_3$  component together with compatible stable oxides of  $\text{ZrO}_2$  or YAG combine many of the advantages of sapphire fiber with morphological stability at elevated temperatures and relative ease of production. Of these the  $\text{Al}_2\text{O}_3$ ,  $\text{ZrO}_2$  system eutectic, with additional 4.2 mol%  $\text{Y}_2\text{O}_3$  modification to create cubic  $\text{ZrO}_2$ , has received considerable attention.<sup>9</sup> This eutectic has demonstrated some attractive characteristics. In the laser heated float zone (LHFZ)-produced eutectic in the form of fibers or slender rods the  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$ <sup>#1</sup> eutectics possess a sub-micron scale morphology in which the majority  $\text{Al}_2\text{O}_3$  phase maintains a topologically continuous form with a remarkably tight [000 1] growth texture.<sup>10,11,14</sup> The c- $\text{ZrO}_2$  phase, in turn, in various morphological shapes ranging from aligned fibers or platelets in either well organized colonies or in less well ordered forms, usually has one narrow dimension in the 0.2  $\mu\text{m}$  range, and is always fully encapsulated in the  $\text{Al}_2\text{O}_3$  phase. This eutectic has fairly reproducible and reasonably attractive levels of fracture toughness<sup>12</sup> and possesses high quality nearly coherent interfaces.<sup>13</sup>

The eutectics contain significant levels of residual stress at room temperature, resulting from thermal misfit between the component phases. The  $\text{Al}_2\text{O}_3$  was found to be in a state uniaxial tension parallel to the axis, at a level of 1.0 GPa while the c- $\text{ZrO}_2$  was in some form of multiaxial compression, roughly at the same level.<sup>10</sup> Such residual stresses that decrease in level at elevated temperatures could still be of importance in initial creep response<sup>14</sup> where they, however, should be rapidly relaxed on the onset of steady creep flow.

Here we will be primarily interested in the creep resistance of these  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2$  eutectics in the 1200–1520 °C range.

## 2. Experimental details

### 2.1. Material and characterization

The eutectics of  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  have all been produced by the Laser Heated Float Zone (LHFZ) method at the NASA John Glenn Research Center.<sup>9</sup> Source rods were

prepared using 99.999% pure polycrystalline  $\text{Al}_2\text{O}_3$  powder (CERAC/pure, Ceralox Corp., Tucson, AZ 08676), and 99.999% pure  $\text{ZrO}_2$  and  $\text{Y}_2\text{O}_3$  powders (Alfa Aesar). These powders were blended in acetone for 70 h using high purity  $\text{Y}_2\text{O}_3$  stabilized  $\text{ZrO}_2$ . The slurry was dried, mixed again and formed into cylindrical rods using cold isostatic pressing. These rods were then sintered at 1500 °C in air for 4 h. A LHFZ process was used to produce the directional solidification of rods as described previously.<sup>2</sup> These rods were re-used as source rods to produce final samples. The objective of the second solidification was to minimize the entrapment of gases at the liquid solid interface. The directional solidification rate was at 2 mm/min for both processes. The directionally solidified (DS) rods had cross-sectional dimensions roughly in the range of 1–1.5 mm diameter changing from rod to rod but showed only small thickness variations along their 20–22 cm length in any one rod. They were composed of 67%  $\text{Al}_2\text{O}_3$  and 33% c- $\text{ZrO}_2$  by volume. Random sectioning of the rods always showed a certain residual component of  $\text{Y}_3\text{Al}_5\text{O}_{12}$  (YAG) at levels that were considered to be insignificant. More significant, however, was an unavoidable level of porosity along the center line of many samples, often with substantial pore dimensions to constitute super-critical flaws for fracture. These pores always had smooth surfaces indicating they resulted from inadequate wetting during melting and solidification of the initial charge. In a few instances they resulted in fracture during loading of the creep experiments where they were given no further attention other than characterization of the pore sizes and shapes as a feed-back to improve the production process. TEM specimens made from randomly spaced axial and transverse sections were used for electron diffraction to check orientation of components. These, as well as a limited number of X-ray pole figure determinations obtained from surfaces of axial and transverse section of the bars demonstrated that the  $\text{Al}_2\text{O}_3$  phase had a growth texture of [000 1] within 2–3 ° parallel to the specimen axis. Wherever prominent three-fold symmetrical colony structures of c- $\text{ZrO}_2$  were observable on transverse sections, there were notable angular differences between the symmetry axes of these colonies indicating the presence of small angle tilt boundaries in the  $\text{Al}_2\text{O}_3$  (often in the range of 10 °) with tilt axes parallel to the growth axis. Since such tilt boundaries would not be stressed in samples under tension parallel to the bar axis, they were not considered to be of any importance in the creep behavior.

A few direct lattice imaging observations were also made of the structure of interfaces. Fig. 1 shows one such direct lattice image of a typical interface that is incoherent but atomically narrow.

Corresponding determination of the texture of the c- $\text{ZrO}_2$  phase, largely based on electron diffraction information, indicated that this phase had primarily a (1 1 2) growth texture with fiber symmetry, incorporating random rotations about this axis.<sup>#2</sup> The sketch of Fig. 2 shows pictorially

<sup>#1</sup> We note that about 4.2 mol% of  $\text{Y}_2\text{O}_3$  in the entire eutectic is required to form cubic zirconia. Hereafter in the paper, however, we refer to cubic zirconia merely as c- $\text{ZrO}_2$ .

<sup>#2</sup> Textures such as (1 1 1) have also been reported by other investigators.

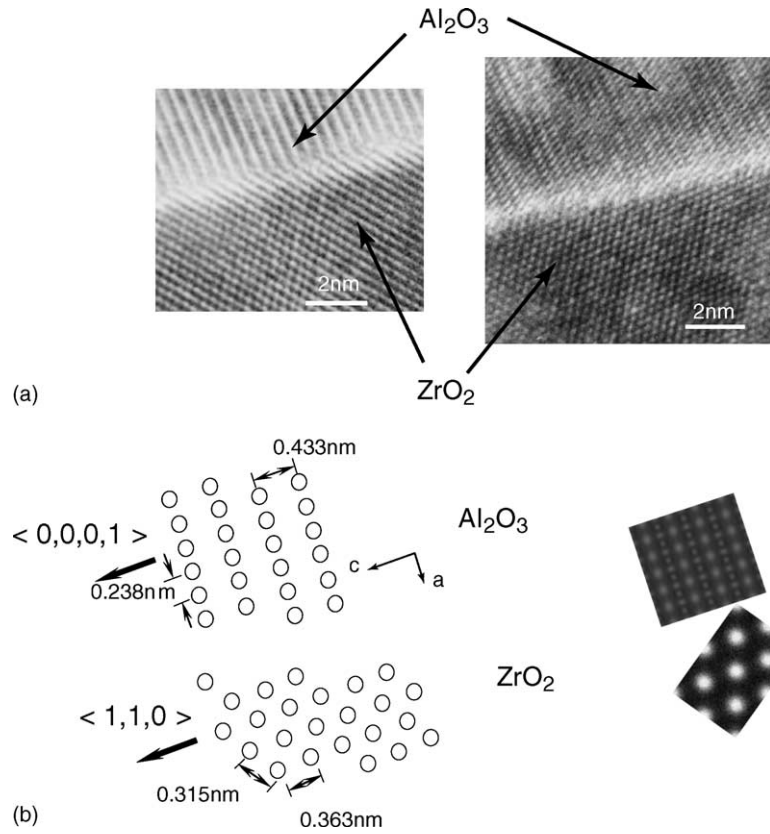


Fig. 1. Direct lattice images of a typical interface between  $\text{Al}_2\text{O}_3$  and  $c\text{-ZrO}_2$  in an  $\text{Al}_2\text{O}_3/c\text{-ZrO}_2(\text{Y}_2\text{O}_3)$  eutectic.

the mutual orientational association of the two component phases.

### 2.2. Creep equipment and creep strain measurement

The creep experiments were carried out in tension inside a Centorr vacuum chamber containing a specially designed

hot zone of 5 cm diameter and 5 cm length made up of a split cylindrical configuration of a 1 mm thick Ta sheet with alternating up-and-down slits to increase the path length of the heating element. The hot zone was surrounded by a series of Mo radiation shields. The long specimens threaded through the hot zone and were gripped at their ends by specially produced metallic friction grips, clamping down on the ends of the specimens through annealed soft nickel sheets to avoid high local contact forces on the rods that could have resulted in fracture. Two relatively massive water cooled OFHC Cu plates were placed between the two exits of the hot zone and the specimen grips to assure that the gripping conditions were, as much as possible, at low temperature not to exceed about 200 °C to avoid slippage in the grips. Stressing of specimens was by externally applied dead-loads.

An optical grade sapphire window permitted viewing the hot specimen through a narrow axial slit in the heating elements to permit direct measurements of the specimen temperature by means of a two-wave-length pyrometer (from Omega Vanzetti, Sharon, MA, USA), providing emissivity-independent measurements. In addition to a control thermocouple inside the hot zone, another thermocouple placed close to the specimen without touching it, was used to actually record the steady temperature in the thermal cavity. The temperature measured by this second thermocouple and that measured by the pyrometer on the specimen usually

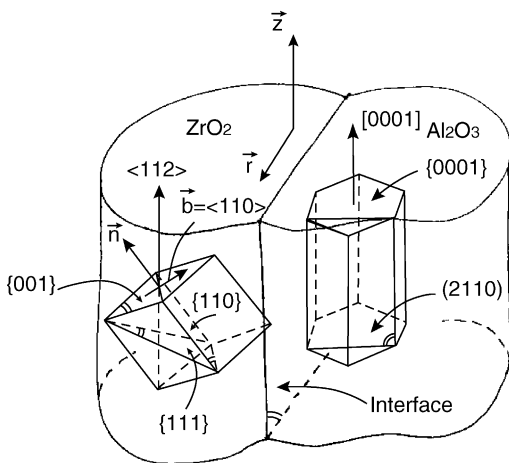


Fig. 2. Sketch showing the textural arrangement of the  $\text{Al}_2\text{O}_3$  component with a preferred  $[000\ 1]$  axis and  $c\text{-ZrO}_2$  with a preferred  $(1\ 1\ 2)$  axis found by electron diffraction analysis. Other textures for  $c\text{-ZrO}_2$  have also been reported.

agreed quite well. The hot zone temperature was automatically controlled by the PID control system of the Centorr equipment.

Since the steady state creep extensions of the specimens under stress were generally quite small, to eliminate random flexures and motions of the massive creep frame from influencing the measurements, the displacements of the two ends of the load train immediately outside the vacuum chamber were simultaneously measured by sets of four LVDTs at both the top and bottom ends of the Centorr chamber. The differences between these two measurements then corresponded to the extension of the specimen. Since the temperature of the specimen inside the hot zone was higher by 1200–1300 °C above the portions of the sample outside the hot zone, the gauge length was taken as the axial extent of the hot zone. Even under ideal conditions the total creep strain was never too uniquely determinable. To overcome this difficulty all creep strains at steady state were always measured incrementally at a given temperature under two different applied stresses and often as loading and unloading cycles. This practice of relative measurements of creep strain did not only give more reliable determinations of the strain rates at the two different levels of stress but also demonstrated the nearly complete absence of transients in such incremental changes in steady state creep. A typical response of this type is shown in Fig. 3 for creep at 1520 °C at stress levels of 150 and 200 MPa. The random-appearing irregularities on the creep curves in Fig. 3 were attributed to be of instrumental origin. When determining actual increments of creep strain in any interval of time these variations were operationally smoothed out. The accuracy of strain measurement was around  $1.4 \times 10^{-5}$ .

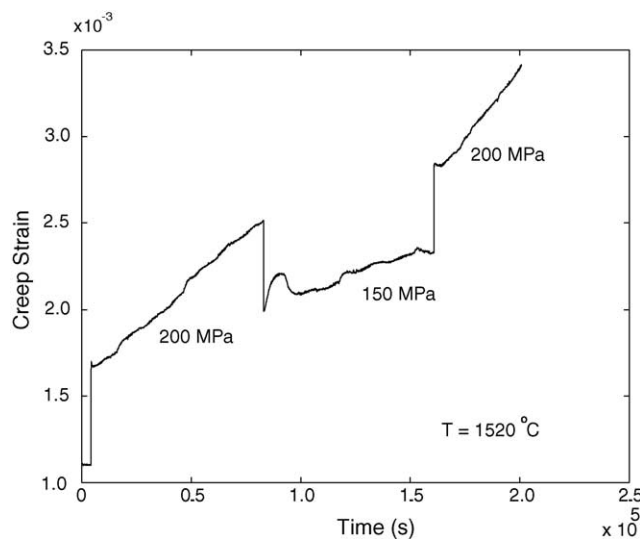


Fig. 3. A typical strain time plot for steady state creep at two different stress levels in an  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2$  eutectic rod at 1520 °C.

### 3. Experimental results

#### 3.1. Transient effects

Upon initial application of stress to a virgin specimen a strain transient was always observed. One such transient recorded for an experiment at 1200 °C under a stress of 300 MPa is shown in Fig. 4. The characteristic time constant of the transient decreased with increasing temperature, e.g. from ca.  $3.4 \times 10^3$  s at 1200 °C to 200 s at 1400 °C. The usual explanation of the transient to be due to initial work hardening in the creeping components prior to establishment of a steady state by recovery processes, familiar in creep of homogeneous metals, was discounted here since no prominent transients were found following stress increases at steady state creep. The most likely cause of the transient was the rapid stress relaxation in the coarse c-ZrO<sub>2</sub> component by creep since for the measured texture of that component there is an abundance of the  $\langle 110 \rangle\{100\}$  principal slip systems that are well oriented for slip, and the creep resistance of c-ZrO<sub>2</sub> in this temperature range is known to be poor.<sup>15</sup> In more ideal morphologies, as presented by Sayir and Farmer<sup>2</sup> and Argon et al.,<sup>17</sup> the coarse fraction of the c-ZrO<sub>2</sub> is located on colony borders. In the usual less ideal morphologies where fibrillar c-ZrO<sub>2</sub> is still aggregated but does not form well-structured colonies the coarse fraction was found to be less regularly distributed (see Fig. 7). An upper bound manifestation of the poor creep resistance of c-ZrO<sub>2</sub> can be assessed from a finite element (FEM) study that will be presented in Section 4.3. There it was found that for the complete relaxation of the deviatoric stresses in the entire c-ZrO<sub>2</sub> phase the volume average deviatoric stresses as well as mean normal stresses in the  $\text{Al}_2\text{O}_3$ , increase roughly by 25–30% which would result in an increment of additional elastic strain in the sample of about  $6 \times 10^{-4}$ . This level is illustrated in Fig. 4

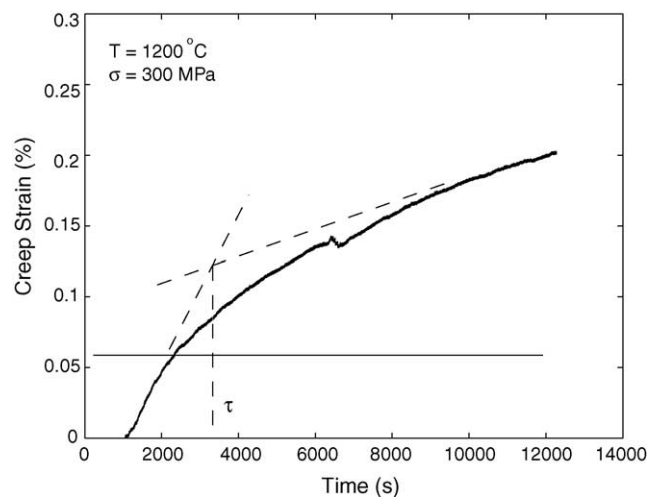


Fig. 4. A typical creep strain transient in an  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2$  eutectic under a stress of 200 MPa at a temperature of 1200 °C. The horizontal line gives an estimate of creep strain increment due to stress relaxation in the coarse fraction of c-ZrO<sub>2</sub>.

Table 1

Recorded steady state strain rates

Stress (MPa)	Strain rate $\dot{\epsilon}$ (sample #)[ $\dot{\epsilon}$ average rate] ( $s^{-1}$ )
1200 °C	
200	$1.97 \times 10^{-10}$ (142-18)
1400 °C	
200	$3.6 \times 10^{-9}$ (187-29)
225	$2.26 \times 10^{-9}$ (187-27-2); $8.16 \times 10^{-9}$ (187-27-1) [ $5.21 \times 10^{-9}$ ]
250	$5.53 \times 10^{-9}$ (187-29)
275	$9.95 \times 10^{-9}$ (187-27-2); $1.10 \times 10^{-8}$ (187-27-1) [ $1.048 \times 10^{-8}$ ]
1520 °C	
150	$1.57 \times 10^{-9}$ (142-16); $4.40 \times 10^{-9}$ (142-16) [ $2.98 \times 10^{-9}$ ]
200	$1.20 \times 10^{-8}$ (142-16); $1.60 \times 10^{-8}$ (142-16) [ $1.40 \times 10^{-8}$ ]

by the horizontal line. The actual amount of stress relaxation in the c-ZrO<sub>2</sub>, however, is difficult to determine since a substantial fraction of this component has phase dimensions in the range of 0.2–0.4 μm and is likely to be dislocation-free and incapable of plastically deforming as we discuss in Section 4. These, and the difficulties in determining the absolute measures of strain derived from the hot zone portions of the samples requires us to de-emphasize this portion of the creep response.

### 3.2. Steady state creep

As already mentioned in Section 2.2 the main information on steady state creep was obtained from incremental experiments of sudden stress increases and decreases at constant temperature as shown in the typical case of Fig. 3 of creep response. A simple check of the magnitude of the instantaneous stretches or contractions for stress increases or decreases, utilizing the appropriate information on the temperature dependent Young's modulus of Al<sub>2</sub>O<sub>3</sub><sup>16</sup> gave in all cases that the recorded jumps were all about a factor of 2 larger than what could be expected from the sample in the hot zone. Since this was well within the additional flexures and relaxations in the load train the recorded strain rates were considered to be reliable.

After the final configuration of the hot zone discussed in Section 2.2 was completed, 11 determinations of steady state creep rate were made, all together at 3 temperatures of 1200, 1400, 1520 °C at stress levels ranging from 150 to 275 MPa, derived from incremental experiments similar to those in Fig. 3. These measurements are listed in Table 1 and are plotted in Fig. 5 together with the creep rates at 1400 °C due to Sayir and Farmer.<sup>2,17,#3</sup> The stress exponents of the

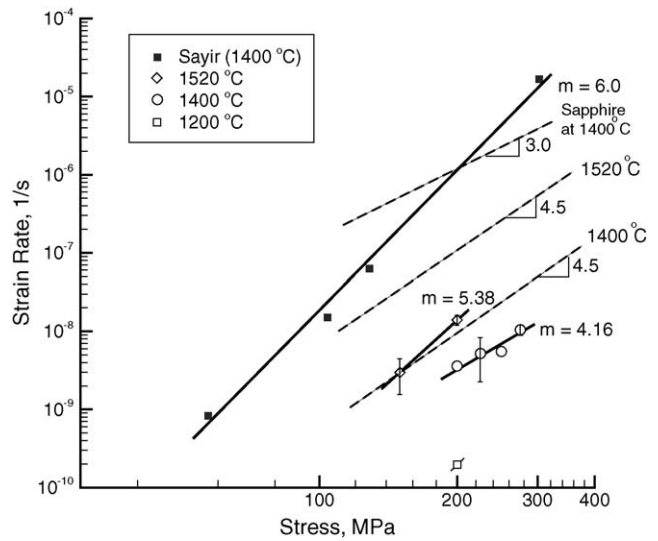


Fig. 5. Plots of steady state creep rates as a function of stress for temperatures of 1200, 1400 and 1520 °C compared with measurements reported by Sayir and Farmer.<sup>2</sup> Broken lines represent predictions of creep model for Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> at 1400 and 1520 °C, together with model prediction for sapphire at 1400 °C.

creep rates at 1400 and 1520 °C, shown in Fig. 5, are 4.16 and 5.38 and are lower than that of Sayir at 6.00. As we will discuss in the creep model in Section 4.3, these exponents are relatively high and need to be explained, as we will do later in Section 4.4.

Table 1 contains also sufficient information for the determination of the activation energy of the governing creep process. The plot of the creep rates at a stress of 200 MPa for three temperatures of 1200, 1400 and 1520 °C in Fig. 6 gives an activation energy of  $Q = 71.1$  kcal/mol for the rate controlling

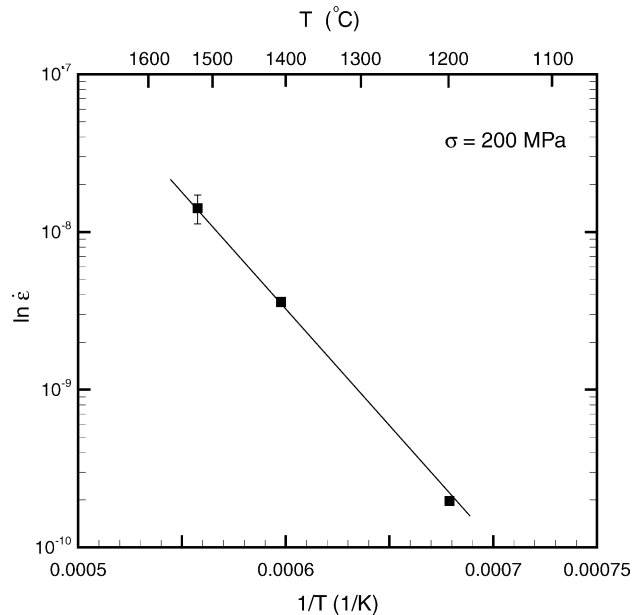


Fig. 6. Determination of activation energy of steady state creep from temperature dependence of steady state creep rate.

<sup>#3</sup> The higher creep rates and the larger stress exponents in these early investigations are most likely a result of not reaching steady state but still being in a late stage of transient behavior.



process in steady state creep which we consider to be due to oxygen ion diffusion which is known to be the slowest species in  $\text{Al}_2\text{O}_3$ . This value is to be compared with 105 kcal/mol for temperatures above 1400 °C and 26 kcal/mol for temperatures below 1400 °C for the same process measured by Oishi and Kingery<sup>25</sup> in pure polycrystalline  $\text{Al}_2\text{O}_3$ . The somewhat lower magnitude of the measured activation energy for oxygen ion diffusion is attributed to the likelihood that a significant fraction of the diffusion current will be along the incoherent interfaces.

#### 4. The creep model

##### 4.1. Basic assumptions

The fact that the *c*- $\text{ZrO}_2$  of considerably lower creep resistance is encapsulated everywhere by the topologically continuous  $\text{Al}_2\text{O}_3$  with a very tight growth texture of [000 1] along the axes of the DS eutectic rods indicates that steady state creep must be controlled by the  $\text{Al}_2\text{O}_3$  phase, once any limited compliance increment due to the stress relaxation in the coarse fraction of *c*- $\text{ZrO}_2$  is complete. This suggests that the overall creep response must have the same characteristics of steady state creep in sapphire single crystals reported by Firestone and Heuer,<sup>3</sup> albeit with certain important added complexities derived from the presence of the *c*- $\text{ZrO}_2$  phase. Fig. 7 shows the well-known phase morphology of a typical  $\text{Al}_2\text{O}_3$ /*c*- $\text{ZrO}_2$  eutectic where the bright phase is the *c*- $\text{ZrO}_2$ . Very much like the  $\gamma'$  phase in superalloy single crystals of CMSX-3,<sup>18</sup> a large fraction of the *c*- $\text{ZrO}_2$  has dimensions in the range of 0.2–0.4  $\mu\text{m}$  and must be regarded as be-

ing too small to have the capability of undergoing plastic flow by independent internal processes of dislocation multiplication by glide or climb, as already remarked above. The *c*- $\text{ZrO}_2$  phase components of larger dimensions, as the irregular shaped ones in Fig. 7 should undergo rapid stress relaxation in the 1200–1520 °C temperature range of interest, as the transient creep curve of Fig. 4 suggests. Such stress relaxation by glide on the {1 0 0}<1 1 0> primary systems in the *c*- $\text{ZrO}_2$  component having a (1 1 2) texture should be relatively unhindered. Even so, as we present in Section 4.3, FEM analysis shows that complete shear stress relaxation in the *c*- $\text{ZrO}_2$  still leaves the mean normal stress intact in it which prevents large average stress enhancement in the  $\text{Al}_2\text{O}_3$  phase.

Since the best slip systems of the basal and prismatic type are largely unstressed for glide due to the tight texture of the  $\text{Al}_2\text{O}_3$  phase, and since the pyramidal dislocations of  $1/3\langle\bar{1}101\rangle$  type are sessile in glide, due to a unique core restructuring process as has been recently demonstrated,<sup>4,5</sup> we expect that creep in this component for eutectic fibers in tension, can only be a consequence of climb of the  $(1/3)\langle\bar{1}101\rangle$  dislocations, whether they exist on the prism planes or on the pyramidal planes. This has already been recognized by Firestone and Heuer.<sup>3</sup> Creep, derived entirely from climbing edge dislocations is quite rare in structural metals. The only previously reported similar case is in Mg single crystals stressed in the [000 1] direction.<sup>6</sup> Nabarro<sup>19</sup> has presented an idealized model of creep derived from climb of a Frank type dislocation network where material fluxes are between the actual climbing dislocations with Burgers vectors parallel to the stress axis and those of other types experiencing no climb forces. Firestone and Heuer<sup>3</sup> found rea-

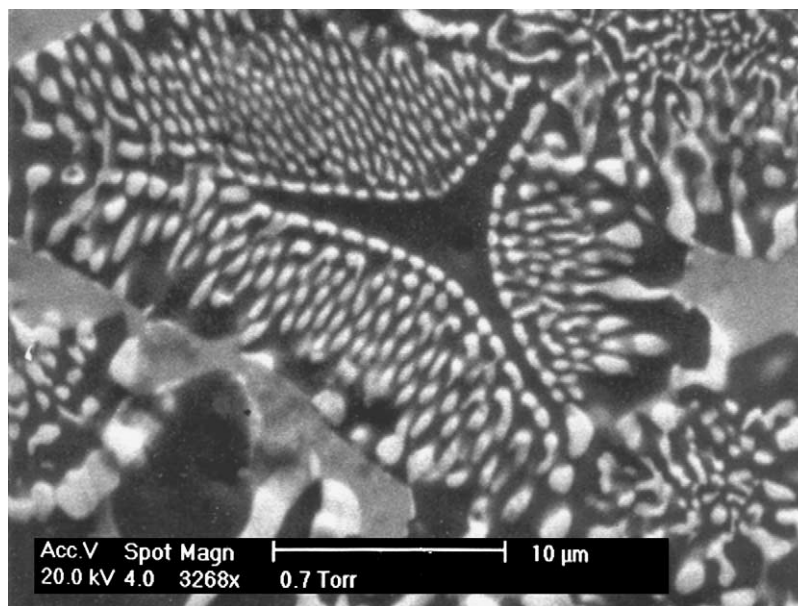


Fig. 7. An SEM micrograph of a typical morphology of the  $\text{Al}_2\text{O}_3$ /*c*- $\text{ZrO}_2$  eutectic, as revealed in a transverse section. The phase morphology of the longitudinal section, while somewhat different, is statistically quite similar to that of the transverse section. The black component is  $\text{Al}_2\text{O}_3$ . Much of the *c*- $\text{ZrO}_2$  component (the bright regions) is of low sub-micron size and is expected not to deform by any form of crystal plasticity.

sonable agreement of their results where the stress exponents were in the range of 3 as the Nabarro model predicts. In our case where the stress exponents are in the range of 4.5–6.0 we expect different conditions to hold, requiring considerable modification of the Nabarro framework.

Since basic TEM information is still largely absent to furnish better guidance, in our model, we make a number of essential assumptions. We assume that the  $(1/3)\langle\bar{1}101\rangle$  dislocations pre-exist in the  $\text{Al}_2\text{O}_3$  phase, having been generated during the directional solidification process while the morphology is being established, and will multiply by topological convolution processes in well known ways from sources, and will be available for climb under stress during creep. We expect that the mean free path lengths of climbing dislocations in  $\text{Al}_2\text{O}_3$  will involve several multiples of the inter-phase dimensions, i.e. be of the order of microns, requiring the climbing dislocations to repeatedly bow and straighten-out as they thread through the small  $c\text{-ZrO}_2$  domains that will remain impenetrable to them. Such repetition of transient line shapes will be viewed as the primary source of the elevation of the creep stress exponents to become larger than 3.<sup>20</sup> We assume that the climbing pyramidal dislocations can multiply by topological process a'la Bardeen and Herring.<sup>21</sup> As for the climb of these dislocations, we assume that this will be diffusion controlled with adequate jog concentrations present along the dislocations, these we expect to be produced by nodal emission processes from network junctions rather than being of a thermal equilibrium nature which preliminary analyses show to be far too difficult. We assume that the diffusional transport will be between dislocation cores and the large concentrations of incoherent or semi-coherent interfaces. We assume also that the interfaces will be completely opaque to the transmission of dislocations from  $c\text{-ZrO}_2$  to  $\text{Al}_2\text{O}_3$  or vice versa. Analysis indicates that such transmission would be subject to extremely large energy barriers, that can not be overcome. We expect that the residual stresses due to initial thermal misfit between the components will be relieved during the transient phases of the creep and provide another contribution to the recorded amounts shown in Fig. 4, but that steady state creep will be governed by the climb forces resulting entirely from the applied stress.

Finally, we expect that as the topologically continuous framework of the  $\text{Al}_2\text{O}_3$  creeps and extends, while the  $c\text{-ZrO}_2$  largely remains either dormant or is merely stress-relaxed, global back stresses will develop in the  $\text{Al}_2\text{O}_3$ , gradually slowing down the creep rate.<sup>22</sup> Since the creep ductilities of the eutectics are quite modest, we do not expect this back stress to develop fast enough to be of consequence, and ignore it.

#### 4.2. The creep rate

Consider a round tensile creep bar of an  $\text{Al}_2\text{O}_3/c\text{-ZrO}_2$  eutectic as sketched in Fig. 8 where local axes 1, 2, 3, are

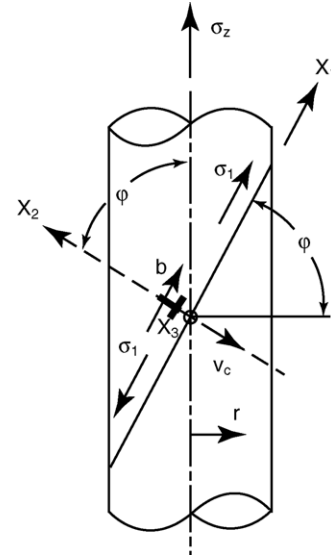


Fig. 8. Sketch of a round bar of a  $\text{Al}_2\text{O}_3/c\text{-ZrO}_2$  eutectic with  $z$ -axis parallel to the  $[0001]$  direction of the  $\text{Al}_2\text{O}_3$ . The plane at angle  $\phi$  outlines a pyramidal glide plane. A local tensile stress  $\sigma_1$  will make a positive edge dislocation climb in the negative  $X_2$ -direction.

chosen in a pyramidal glide system with axis 1 chosen parallel to the  $1/3\langle\bar{1}101\rangle$  Burgers vector, axis 2 normal to the pyramidal plane making an angle  $\phi$  with the bar axis, and axis 3 parallel to the line vector of a positive edge dislocation. Under a uniform stress  $\sigma_1$  a positive edge dislocation will climb in the negative 2 direction with a velocity  $v_c$  to contribute to a uniaxial tensile creep strain rate parallel to the 1 direction. This velocity, if it were governed by diffusion controlled climb would be given by Friedel<sup>23</sup>:

$$v_c = \frac{2\pi D(\sigma_1 - \sigma_T)b^2}{kT \ln(r_s/r_c)} \quad (1)$$

where  $D = D_0 \exp(-Q/RT)$  is the diffusion constant of  $\text{Al}_2\text{O}_3$  where oxygen is recognized to be the slow diffusing species,  $\sigma_T$  is the triaxial component of the applied stress ( $\sigma_z/3$ ) that does not promote equilibrium climb,  $r_s$  is the distance from the dislocation core to vacancy sinks along the interfaces,  $r_c$  the core radius from which point defect emission is considered,  $b$  the magnitude of the Burgers vector,  $D_0$  the pre-exponential factor of the diffusion constant,  $Q$  the activation energy of O ion diffusion in  $\text{Al}_2\text{O}_3$  and  $R$  and  $T$  have their usual meaning. The kinematics of shape change due to climb alone of dislocations has been considered more broadly by Groves and Kelly.<sup>24</sup> In our case a more specialized consideration will be adequate. Thus, considering that there will be three equivalent pyramidal systems that can contribute to the axial strain rate equally, as is explained in the Appendix A, the axial creep strain rate  $\dot{\epsilon}_z$  along the bar and the accompanying radial contractile strain rates  $\dot{\epsilon}_r$  are given respectively as:

$$\dot{\epsilon}_z = (2 - 3 \cos^2 \phi) \dot{\epsilon}_0 \quad (2a)$$

and

$$\dot{\varepsilon}_r = - \left( 1 - \frac{3}{2} \cos^2 \phi \right) \dot{\varepsilon}_0 \quad (2b)$$

where

$$\dot{\varepsilon}_0 = b \rho_m v_c \quad (3)$$

is the main creep rate in the pyramidal system coordinates without regard to overall volume preservation, which is considered to result in Eqs. (2) and (3) (see Appendix A). Relating the actual climb-producing tensile stress  $\sigma_1$  to the axial stress  $\sigma_z$  and considering the volume-average effect of the local variation of stress due to the presence of the c-ZrO<sub>2</sub> components by a factor  $q$  we have

$$\sigma_1 - \sigma_T = \frac{2}{3} q \sigma_z (1 - \cos^2 \phi) \quad (4)$$

If the mobile dislocation density,  $\rho_m$ , of climbing dislocations at steady state is governed by mutual interactions in a self adjusting basis<sup>19</sup>; i.e.

$$\rho_m \cong \left( \frac{2\pi\sigma_z}{\mu b} \right)^2 q^2 (1 - \cos^2 \phi)^2 \quad (5)$$

we have, by finally combining Eqs. (2), (4), (5) and (6) the axial steady state creep rate:

$$\dot{\varepsilon}_z = A \left( \frac{D}{b^2} \right) \left( \frac{\sigma_z}{\mu} \right)^3 \left( \frac{\mu \Omega}{kT} \right) \frac{(1 - \cos^2 \phi)^3 (2 - 3 \cos^2 \phi)}{\ln(r_s/r_c)} \quad (6)$$

where

$$A = \frac{2}{3} (2\pi)^3 \frac{q^3}{\beta} \quad (7a)$$

and

$$\beta = \Omega/b^3 = 0.079 \quad (7b)$$

where  $\Omega$  is the ionic volume of O in the Al<sub>2</sub>O<sub>3</sub> lattice and  $q$  is a factor which relates the local volume average climb stress to the axial stress  $\sigma_z$  as is determined from a FEM analysis of the stress distribution in the Al<sub>2</sub>O<sub>3</sub> phase under an applied tensile stress as discussed in Section 4.3 (Fig. 9).

Eq. (7) gives the creep rate due to the climb of quasi-straight dislocations in a homogeneous stress field. As we discuss in Section 4.4 this is not the case in the Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> eutectics where the climbing dislocations need to thread through the non-deforming c-ZrO<sub>2</sub> domains acting as dispersoids and are required to alternately bow around these domains and be released to straighten out, acting effectively as if straight dislocations were moving through a strongly varying internal stress field which will increase the stress exponent in predictable ways and decrease the actual creep rate.<sup>20</sup>

In Section 4.5 we will evaluate the creep model and compare it with the experimental results.

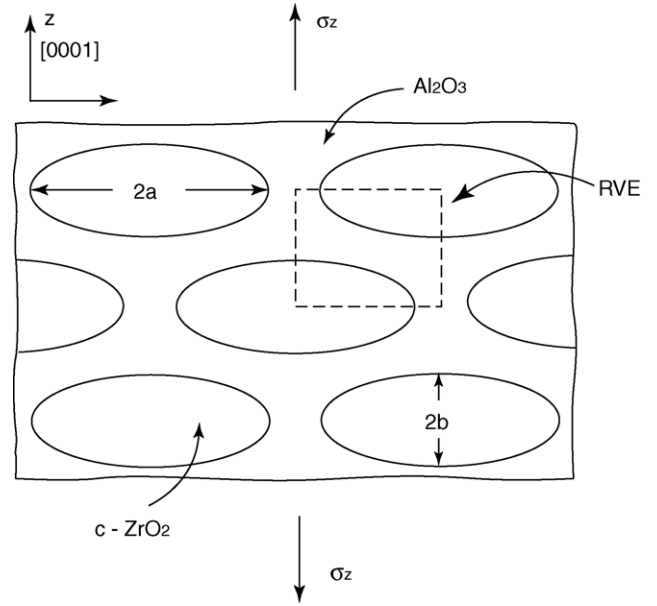


Fig. 9. Sketches of an idealization of the c-ZrO<sub>2</sub> domains as ellipsoidal cylinders in the topologically continuous Al<sub>2</sub>O<sub>3</sub> component, for the purpose of a FEM study of the internal stress distribution in the Al<sub>2</sub>O<sub>3</sub> when a tensile stress  $\sigma_z$  is applied.

#### 4.3. Distribution of stresses in the Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> eutectics

A very important way in which creep in the eutectics differs from creep in single crystal sapphire is that in the former the stresses are distributed in a complex manner. In addition to residual stresses arising from the different thermal expansions of the two components, the applied stresses result in complex internal local stress distributions due to different elastic properties of the Al<sub>2</sub>O<sub>3</sub> and the c-ZrO<sub>2</sub> component phases. Thus, to develop some necessary understanding of this phenomenon on a broad basis, a linear-elastic FEM analysis was carried out in a 2-D plane strain setting of the eutectic in the ordered regions shown in Fig. 7. Here the c-ZrO<sub>2</sub> domains in 2-D are taken as elliptical cylinders occupying a volume fraction of 0.33. The chosen representative computational volume is indicated in this figure. We have considered the c-ZrO<sub>2</sub> to be always fully relaxed of all shear stresses but with an unrelaxed appropriate bulk modulus of 83 GPa. This is accomplished operationally in the FEM code for purely elastic behavior by choosing the Young's modulus of the c-ZrO<sub>2</sub> as 0.05 MPa and the Poisson's ratio as 0.4999999. All other chosen material constants are listed in Table 2. In the broader analysis we considered the aspect ratios of ellipsoidal rods,  $a/b$  of c-ZrO<sub>2</sub> as 1, 3, 4 and 5. Of these results we present here only the distributions for  $a/b = 3.0$  as most representative. All analyses were limited to the elastic range. In this FEM framework we considered several process simulations: (a) an applied tensile stress  $\sigma_z$  alone; (b) cooling the morphology from a stress-free condition at 1875 °C down



Table 2  
Material properties for linear elastic FEM analysis

	Al <sub>2</sub> O <sub>3</sub>	c-ZrO <sub>2</sub>
Young's modulus	190 GPa at 1400 °C	0.05 MPa at 1400 °C <sup>a</sup>
Poisson's ratio	0.3	0.4999999 <sup>a</sup>
Yield stress	5.0 GPa <sup>b</sup>	5.0 GPa
Coefficient of thermal expansion (°C <sup>-1</sup> )	9.1 × 10 <sup>-6</sup> (a) 9.9 × 10 <sup>-6</sup> (c) <sup>14</sup>	12.9 × 10 <sup>-6</sup> <sup>14</sup>

<sup>a</sup> These values were chosen in the FEM code result in a negligible shear modulus for c-ZrO<sub>2</sub> to obtain a state of near complete shear relaxation while leaving the bulk behavior intact.

<sup>b</sup> These high yield stresses were chosen in the FEM code to guarantee pure elastic response.

to 1400 °C to determine residual stresses and; (c) cooling as in (b) plus an applied tensile stress of  $\sigma_z = 300$  MPa. Of these, we discuss here only the case of the internal stress distribution under an applied stress  $\sigma_z = 300$  MPa (since behavior is linear-elastic, results for all other stress levels can be determined by re-scaling).

In Fig. 10a we show the distribution of the Mises deviatoric stresses for the case of  $\sigma_z = 300$  MPa and  $T = 1400$  °C, in the Al<sub>2</sub>O<sub>3</sub> region. As expected, the states of stress inside the ellipsoidal regions of the c-ZrO<sub>2</sub> are close to constant. Fig. 10b gives the more important distribution of the climb stress  $\sigma_1$ , parallel to the pyramidal dislocation Burgers vector at an angle of  $\phi = 57.7^\circ$  as shown in Fig. 8. The volume-average level of this important climb producing stress was found to be  $\sigma_1 = 216.3$  MPa which gives the factor  $q$  to be 0.721. This value appears surprisingly low in view of the assumed complete relaxation of the deviatoric stresses in the c-ZrO<sub>2</sub>, until it is recognized that the c-ZrO<sub>2</sub> still supports fully a mean normal stress which limits the stress enhancement in the Al<sub>2</sub>O<sub>3</sub> component.

#### 4.4. Effect of internal resistance variations

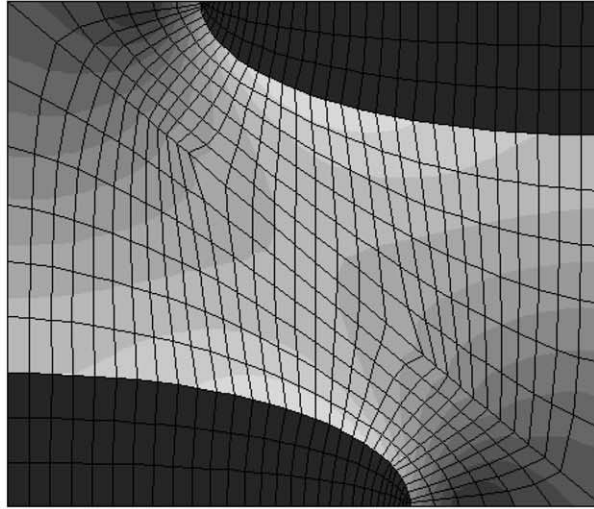
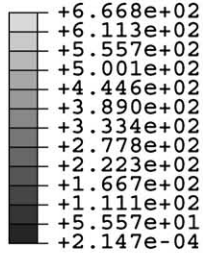
As stated in Section 4.2 the creep response of the Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> eutectics differ significantly from the creep behavior in homogeneous sapphire single crystals studied by Firestone and Heuer.<sup>3</sup> First because of the non-uniform stress distribution in the morphology of Al<sub>2</sub>O<sub>3</sub> and c-ZrO<sub>2</sub> discussed in Section 4.3 above but even more importantly by the large perturbations that the climbing dislocations encounter threading through the isolated and largely non-deforming c-ZrO<sub>2</sub> domains. Thus, consider the convolutions that a climbing dislocation has to go through as it threads through the gaps of the c-ZrO<sub>2</sub> domains as depicted in Fig. 11. First, the climbing dislocation must squeeze into the gaps between the domains as depicted in Fig. 11a to a critical configuration much like the Orowan bowing process for non-shearable dispersoids. The peak stress that is required for this configuration to be achieved is

$$\sigma = \sigma_1 = \frac{2\mathcal{E}}{b\Lambda} = \frac{\mu b}{\Lambda} \quad (8)$$

where  $\mathcal{E}$  is the dislocation line tension ( $\mu b^2/2$ ) and  $\Lambda$  is the size of the inter-domain gap. Clearly, here the process is not

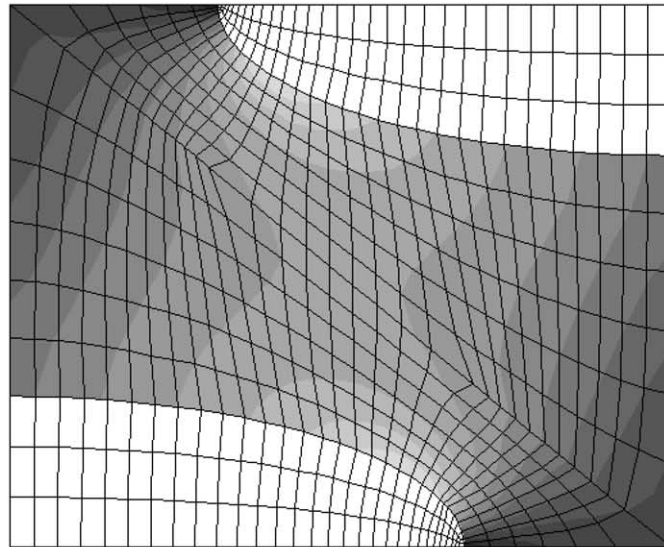
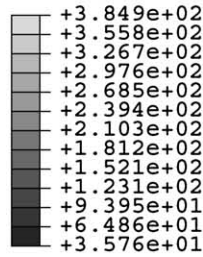
one of glide but climb where  $\sigma$  is the climb stress. Once the critical configuration is reached and the climbing dislocation surrounds the impenetrable domain and pinches off as shown in Fig. 11b, the separated dislocation has acquired a definite cusp where this shape of the dislocation will undergo accelerated climb due to the advantageous line tension effects. We view these required contortions of the dislocations as if they were the same as positive and negative internal stresses  $\sigma_i$ , retarding and then accelerating climb, very much like the corresponding cases of favorable and unfavorable internal stresses that a glide dislocation needs to go through between dispersoids. Such problems were treated in great detail by many investigators, but particularly thoroughly by Li,<sup>20</sup> which illustrated that a gliding dislocation (in our case a climbing dislocation) loses more time in the region of adverse internal stress than the time it gains in moving through regions of favorable internal stress. The effect is illustrated in Fig. 12, where 12a shows the sinusoidally varying internal stress  $\sigma_i$  and the prevailing climb stress  $\sigma$ . The most important consequence of such deceleration and acceleration of motion is an increase in the effective stress exponent  $m$  of the dislocation velocity, scaled by the ratio of  $\sigma_i/\sigma$  is shown in Fig. 12b. A secondary effect is a factor  $C = \bar{v}/v$  that gives the decrease in the average dislocation velocity relative to the velocity in unhindered climb shown in Fig. 12c, also scaled with the ratio of  $\sigma_i/\sigma$ . To assess a measure of the effect we note from Fig. 7 that in much of the morphology the inter-domain distance  $\Lambda$  would appear to be between 0.3 and 1.0  $\mu\text{m}$ . Taking the small dimension to determine the maximum level of this effect, together with  $b = 5.12 \times 10^{-10}$  m and  $\mu = E/2(1 + \nu) = 73$  GPa<sup>16</sup> at 1400 °C we determine  $\sigma_i = 125$  MPa. This, for an applied stress  $\sigma_z = 300$  MPa, in the range of interest, that would give a volume average climb stress  $\sigma = q\sigma_z(1 - \cos^2 \phi)$  of 153 MPa, and a ratio  $\sigma_i/\sigma \approx 0.815$ , Fig. 12b indicates that the effective stress exponent of the stress in the velocity expression should increase to nearly  $m = 3$  from unity as the maximum effect. Moreover, Fig. 12c gives  $C = \bar{v}/v = 0.6$ . Clearly, larger distances  $\Lambda$  should result in smaller effective internal stresses while smaller applied stresses should give increased ratios  $\sigma_i/\sigma$ . The overall net effect will be rather complex to assess. Here, we consider that under the conditions described the net effect on the overall stress exponent of the creep rate will be to elevate it from  $n = 3$  for the model presented above to  $n = (m + 2) = (2.5 + 2) = 4.5$  to  $n = (3 + 2) = 5$ . This consideration and the factor  $C$ , when

S, Mises  
(Ave. Crit.: 75%)



(a)

Resolved Climbing Stress  
(Ave. Crit.: 75%)



(b)

Fig. 10. Stress distribution results of the FEM study: (a) deviatoric (Mises) stresses in the two components of the eutectic and (b) the distribution of the climb stress  $\sigma_1$  in the  $\text{Al}_2\text{O}_3$ .

incorporated into the creep model of smooth climb given by Eq. (7) will change it finally to

$$\dot{\epsilon} = AC \left( \frac{D}{b^2} \right) \left( \frac{\sigma_z}{\mu} \right)^n \left( \frac{\mu \Omega}{kT} \right) \frac{(1 - \cos^2 \phi)^3 (2 - 3 \cos^2 \phi)}{\ln(r_s/r_c)} \quad (9)$$

It is this expression that we will compare with the experimental results in Section 4.5.

#### 4.5. Evaluation of the creep model

We now proceed to evaluate our final creep model of Eq. (9) and compare it with experimental results. In the comparison we will evaluate the expression for both 1400 and 1520 °C. We use the following model parameters and material properties:

$$\mu = \frac{E}{2(1+\nu)} = 73 \text{ GPa at } 1400 \text{ }^\circ\text{C and } 63.4 \text{ GPa at } 1520 \text{ }^\circ\text{C}^{16}$$

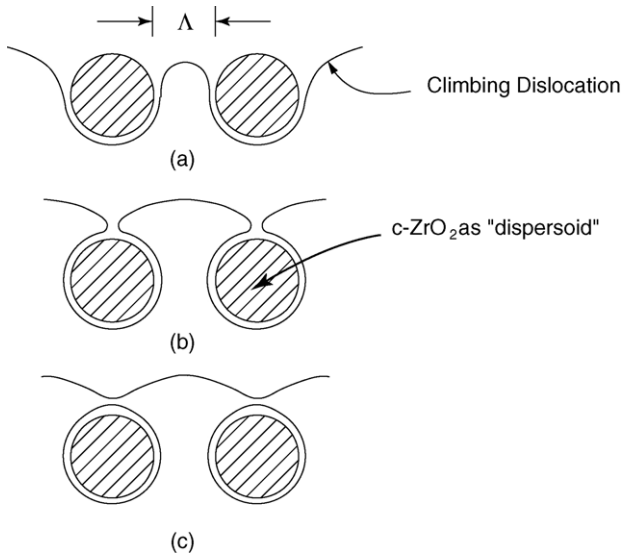


Fig. 11. Sketch depicting the required contortions that a climbing edge dislocation need to suffer in bowing through the gaps between the c-ZrO<sub>2</sub> domains that we consider to be impenetrable: (a) critical climb configuration to bow through the gap between two c-ZrO<sub>2</sub> dispersoids at a spacing  $\Lambda$ , (b) at the point when the climbing dislocation is just about to pinch off, and (c) the cusped dislocation line straightening out under the application of line tension.

$b = 5.12 \times 10^{-10} \text{ m}$   
 $\Omega = 1.06 \times 10^{-29} \text{ m}^3$   
 $q = 0.721$  as determined in Section 4.3

$\beta = \Omega/b^3 = 0.079$   
 $A = \frac{2}{3}(2\pi)^3(q^3/\beta) = 7.85 \times 10^2$   
 $\phi = 57.7^\circ$   
 $r_s = 0.5\text{--}2.0 \text{ }\mu\text{m}$ , as estimated from micrographs (Fig. 7)  
 $r_c \approx b$

$D = 10^{-14} \text{ cm}^2/\text{s}$  at 1400 °C and  $9 \times 10^{-14} \text{ cm}^2/\text{s}$  at 1520 °C determined from Oishi and Kingery directly for their polycrystalline material<sup>25</sup> for which, parenthetically the activation energy was  $Q = 110 \text{ kcal/mol}$ , considerably larger than our value.

Finally, we take  $n = 4.5$  as suggested from our analysis in Section 4.4 above, and

$C = 0.6.$

The calculated steady state creep rate relations for 1400 and 1520 °C are presented in Fig. 5 as the broken lines in comparison with the experimental data.

Considering the several uncertainties in the model and material parameters the agreement between model and experimental results is pleasing.

**5. Discussion**

The experimental measurements of steady state creep rates in the Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> eutectic, while few, have given a good measure of the response of this material in the temperature

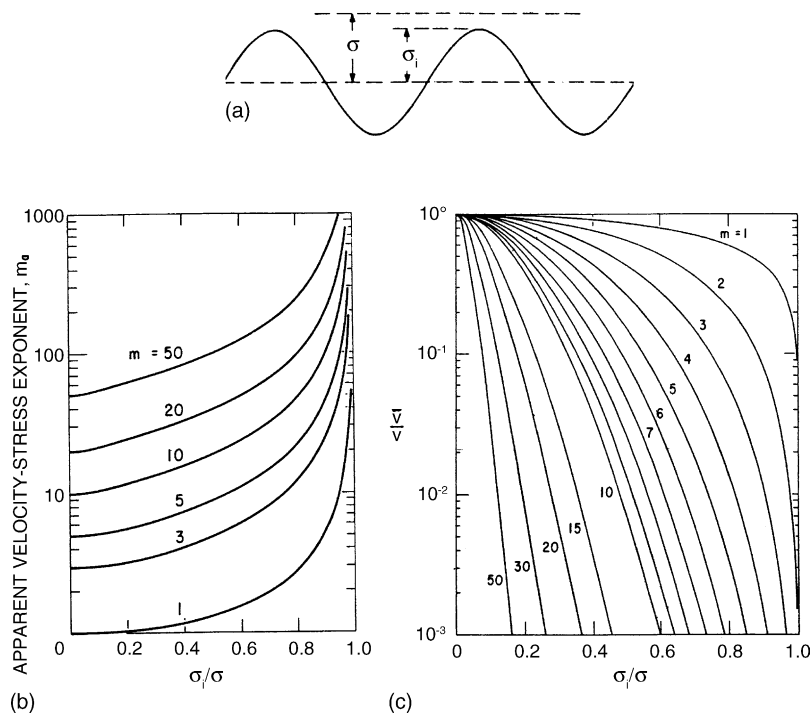


Fig. 12. Consequence of the repeated bowing out and straightening of the climbing dislocation considered as a set of internal resistances  $\sigma_i$  alternately retarding and speeding up the climbing dislocation: (a) retardation and speeding up considered as unfavorable and favorable internal stresses  $\sigma_i$  in the presence of a climb stress  $\sigma$ , (b) effect of  $\sigma_i/\sigma$  on the overall stress exponent  $m$  of the dislocation velocity, and (c) effect of  $\sigma_i/\sigma$  on the attenuation factor  $C$  on the average climb velocity (figures reproduced from Li,<sup>20</sup> courtesy of John Wiley & Sons).

range of interest. Taking account of the possible interactions of the c-ZrO<sub>2</sub> phase with the Al<sub>2</sub>O<sub>3</sub> we have developed a steady state creep model based on climb of pyramidal edge dislocations in Al<sub>2</sub>O<sub>3</sub> alone. In our model we propose that the c-ZrO<sub>2</sub> phase of relatively large size, in the micron range will most likely undergo nearly complete stress relaxation in the 1400 °C range, resulting in an initial transient creep with relatively short time constant of roughly 200 s.<sup>2</sup> On the other hand the sub-micron size domains of the c-ZrO<sub>2</sub> will most likely be too small to undergo independent deformation processes by crystal plasticity and will remain non-deformable to impede the climb motion of the pyramidal system dislocations in the Al<sub>2</sub>O<sub>3</sub> much like dispersoids. The observed creep rate stress exponents in the range of 4–5 are indicative of this behavior. In this sense it would appear that the eutectic through its unique morphology should be more creep resistant than sapphire of [000 1] axis orientation. To explore this comparison we interpret our creep model for application to sapphire in which the climb motion of the dislocations is expected to be quasi-smooth. Such a modification would give a steady state creep expression of

$$\dot{\varepsilon}_{\text{sap}} = A' \left( \frac{D'}{b^2} \right) \left( \frac{\sigma_z}{\mu} \right)^3 \left( \frac{\mu \Omega}{kT} \right) \times \frac{(1 - \cos^2 \phi)^3 (2 - 3 \cos^2 \phi)}{\ln(r_s/r_c)} \quad (10)$$

where  $A' = \frac{2}{3}(2\pi)^3/\beta = 2.09 \times 10^3$  (for  $q = 1.0$ ) and  $D'$ , the diffusion constant, must be chosen for sapphire single crystals not having the benefit of diffusion short circuits along interfaces. From Oishi and Kingery<sup>25</sup> we obtain  $D' = 3.5 \times 10^{-17}$  cm<sup>2</sup>/s at 1400 °C, or a factor of  $3.5 \times 10^{-3}$  lower than for polycrystalline material or for our eutectic with a large volume concentration of interfaces. For these alterations, but for all other factors remaining the same we determine a steady state creep expression for sapphire crystals of [000 1] orientation at 1400 °C which is shown as the dashed line in Fig. 5, confirming our expectation of a lower creep resistance in comparison with the Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub> eutectic.

The creep model we presented is based on the existing evidence of the measured creep rates, their stress and temperature dependence and on the findings of the earlier work of Firestone and Heuer<sup>3</sup> on single crystal sapphire. However, the work leaves many unanswered questions. Foremost among these are the details of the continuance of the all-important fluxes of climbing dislocations, their origins, their form of maintenance and the fine structure of cores of dislocations such as details of jogs where the actual climb steps occur by O ion vacancy emission. Preliminary energetic considerations of dislocation emission from ledges on the ubiquitous interfaces or from misfit dislocations along interfaces have indicated very large energy barriers under the prevailing local stresses. However, other forms of heterogeneous nucleation of dislocations from interfaces must still be consid-

ered. Other possibilities for inelastic behavior such as Coble creep were discounted because of the apparent general absence of transverse boundaries in the topologically continuous Al<sub>2</sub>O<sub>3</sub> component and the generally linear stress dependence of creep flow by this mechanism, which if present, appears to be swamped by dislocation climb flow. Parenthetically, it should be quite likely that the encapsulated c-ZrO<sub>2</sub> components can undergo stress relaxation entirely by diffusional Coble flow along its interfaces with the Al<sub>2</sub>O<sub>3</sub> component, but in the absence of transverse grain boundaries in the Al<sub>2</sub>O<sub>3</sub>, that topologically continuous component can elongate only by dislocation fluxes by climb. Other possible mechanisms as well as the missing details of the maintenance of climb fluxes of dislocations could come from TEM studies which are now in progress, and would be reported elsewhere.

## 6. Conclusions

- (1) In the eutectic of Al<sub>2</sub>O<sub>3</sub>/c-ZrO<sub>2</sub>(Y<sub>2</sub>O<sub>3</sub>) the majority phase of Al<sub>2</sub>O<sub>3</sub> is topologically continuous, has a nearly perfect texture of [000 1] parallel to the growth direction and encapsulates everywhere the minority c-ZrO<sub>2</sub> phase which itself has a growth texture of  $\langle 112 \rangle$ .
- (2) The interfaces separating the phases are well structured but are incoherent over most of the area.
- (3) A large fraction of the c-ZrO<sub>2</sub> phase has a sub-micron oriented fibrillar or plate-like morphology which often, but not always, aggregates into colonies. The remaining fraction, usually surrounding the colonies of oriented fibrils or platelets is of a coarser, micron size.
- (4) Upon first application of stress, transient creep is observed which is attributed to stress relaxation in the coarser fraction of the c-ZrO<sub>2</sub>. The sub-micron fraction is considered to be too small to undergo deformation by crystal plasticity and is expected to be dormant.
- (5) In steady state creep, stress changes do not produce additional transients within the resolution of stress measurements.
- (6) The stress dependence of the steady state creep rate is of power-law form with an exponent in the range between 4.5 to 5.0. The activation energy of the creep rate is 71.1 kcal/mol in the 1400–1500 °C range and is attributed to oxygen ion diffusion through the Al<sub>2</sub>O<sub>3</sub> and along the interfaces.
- (7) Because of the [000 1] texture of the Al<sub>2</sub>O<sub>3</sub> phase which must control the overall creep rate for topological reasons, the proposed steady state model is based on the climb of the  $(1/3)\langle \bar{1}101 \rangle$  dislocations in Al<sub>2</sub>O<sub>3</sub> since the latter are sessile in glide.
- (8) In the creep model which is a generalization of the steady state diffusional creep model of Nabarro,<sup>19</sup> the stress exponents of larger than 3 are attributed to the necessity of repeated bowing and straightening of the climbing dislocations in Al<sub>2</sub>O<sub>3</sub> as they circumvent the sub-micron c-ZrO<sub>2</sub> domains.



- (9) The predictions of the creep model, which has no adjustable constants, agree quite well with experimental observations.
- (10) Finally, a mechanistic comparison of the creep resistance of the  $\text{Al}_2\text{O}_3/\text{c-ZrO}_2(\text{Y}_2\text{O}_3)$  eutectic with that of [0001] sapphire single crystal indicates that the former appears superior because of the obstructions of the dispersoid-like  $\text{c-ZrO}_2$  domains to the the climbing dislocations of the  $\text{Al}_2\text{O}_3$  phase, even though the diffusion constant of sapphire single crystals is three orders of magnitude smaller than that of the eutectic.

### Acknowledgements

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### Appendix A. The kinematics of creep strain rate by climb

As presented in Section 4.2, if a single set of parallel edge dislocations of density  $\rho$  were to climb with a velocity  $v_c$ , a strain rate  $\dot{\epsilon}_0 = b\rho v_c$  would result in the direction perpendicular to the plane of the climbing dislocations.

Referring now to Fig. 8, consider the coordinate axis set 1, 2, 3 of a pyramidal glide system in a round bar of  $\text{Al}_2\text{O}_3$  where the 1 axis is parallel to the Burgers vector of the pyramidal edge dislocations and the 1, 3 pyramidal plane makes an angle  $\phi$  with respect to the bar axis  $z$ . Under a tensile stress  $\sigma_1$  then, a density  $\rho$  of dislocations climbing in the 2 direction (positive edge dislocations in the negative 2 direction and negative edge dislocations in the positive 2 direction) a creep rate  $\dot{\epsilon}_0$  in the 1 direction of would result. However, such climb requires extension of extra half planes which requires material fluxes to arrive to the dislocation from all directions and dispersal of vacancies into all directions, resulting in contractile strain rates  $\dot{\epsilon}_{1c}, \dot{\epsilon}_{2c}, \dot{\epsilon}_{3c}$ . For preservation of volume, it would be necessary that

$$\dot{\epsilon}_0 + \dot{\epsilon}_{1c} + \dot{\epsilon}_{2c} + \dot{\epsilon}_{3c} = 0 \quad (\text{A.1})$$

and since by symmetry

$$\dot{\epsilon}_{1c} = \dot{\epsilon}_{2c} = \dot{\epsilon}_{3c} \quad (\text{A.2})$$

we have net strain rates

$$\dot{\epsilon}_1 = \frac{2}{3}\dot{\epsilon}_0 \quad (\text{A.3a})$$

and

$$\dot{\epsilon}_2 = \dot{\epsilon}_3 = -\frac{1}{3}\dot{\epsilon}_0 \quad (\text{A.3b})$$

Resolving these strain rates into the axes  $r$ , and  $z$  of the creep bar we have

$$\dot{\epsilon}_z = \left(\frac{2}{3} - \cos^2 \phi\right)\dot{\epsilon}_0 \quad (\text{A.4a})$$

$$\therefore \dot{\epsilon}_r = -\left(\frac{1}{3} - \frac{1}{2}\cos^2 \phi\right)\dot{\epsilon}_0 \quad (\text{A.4b})$$

Since there are three sets of pyramidal dislocations, all with equal capacity to contribute to axial strain rate, we take the total axial strain rate to be

$$\dot{\epsilon}_z = (2 - 3\cos^2 \phi)\dot{\epsilon}_0 \quad (\text{A.5a})$$

and

$$\dot{\epsilon}_r = -\left(1 - \frac{3}{2}\cos^2 \phi\right)\dot{\epsilon}_0 \quad (\text{A.5b})$$

where the dislocation velocity in  $\dot{\epsilon}_0$  is still related to tensile stresses parallel to the Burgers vectors of the pyramidal dislocations. Resolving these tensile stresses to the axial tensile stress  $\sigma_z$ , then gives the climb-causing tensile stresses

$$\sigma_1 = q\sigma_z(1 - \cos^2 \phi) \quad (\text{A.6})$$

where the factor  $q$ , which needs to be determined from a FEM boundary value problem, refers to the volume average of the climb producing stress in the  $\text{Al}_2\text{O}_3$  component between the  $\text{c-ZrO}_2$  components. This development is discussed in Section 4.3.

### References

1. Fleischer, R. L., High strength, high temperature intermetallic compounds. *J. Mater. Sci.*, 1987, **22**, 2281–2288.
2. Sayir, A. and Farmer, S. C., The effect of microstructure on mechanical properties of directionally solidified  $\text{Al}_2\text{O}_3/\text{ZrO}_2(\text{Y}_2\text{O}_3)$  eutectic. *Acta Mater.*, 2000, **48**, 4691–4697.
3. Firestone, R. F. and Heuer, A. H., Creep deformation of  $0^\circ$  degree sapphire. *J. Am. Ceram. Soc.*, 1976, **59**, 24–29.
4. Chiang, J., Bodur, C. T. and Argon, A. S., Pyramidal edge dislocation cores in sapphire. *Phil. Mag. Lett.*, 2003, **83**, 659–666.
5. Bodur, C. T., Chiang, J. and Argon, A. S., Molecular dynamics simulations of basal and pyramidal system edge dislocation cores in sapphire. *J. Eur. Ceram. Soc.*, 2005, **25**, 1431–1439.
6. Edelin, G. and Poirier, J. P., A study of dislocation climb by means of diffusional creep experiments in magnesium. I. Deformation mechanism. *Phil. Mag.*, 1973, **28**, 1203–1210 [in French].
7. Morgan, P.-E.-D. and Marshall, D. B., Ceramic composites of monazite and alumina. *J. Am. Ceram. Soc.*, 1995, **78**, 1553–1563.
8. Lev, L. C. and Argon, A. S., Oxide-fiber/oxide-matrix composites. *Mater. Sci. Eng.*, 1995, **A195**, 251–261.
9. Sayir, A., Directional solidification of eutectic ceramics. Unpublished report (NASA Glenn Research Center, private communication).
10. Fraser, C. S., Dickey, E. C. and Sayir, A., Crystallographic texture and orientation variants in  $\text{Al}_2\text{O}_3/\text{Y}_3\text{Al}_5\text{O}_{12}$  directionally solidified eutectic crystals. *J. Cryst. Growth*, 2001, **233**, 187–195.



11. Yi, J., Creep Resistance of Directionally Solidified Eutectic Ceramic: Experiments and Models PhD Thesis, Mech. Eng. Dept. M.I.T., 2004.
12. Pastor, J. Y., Poza, P., Llorca, J., Pena, J. I., Merino, R. I. and Orera, V. M., Mechanical properties of directionally solidified  $\text{Al}_2\text{O}_3/\text{ZrO}_2(\text{Y}_2\text{O}_3)$  eutectics. *Mater. Sci. Eng.*, 2001, **A308**, 241–249.
13. Mazerolles, L., Michol, D. and Portier, R., Interfaces in oriented  $\text{Al}_2\text{O}_3/\text{ZrO}_2(\text{Y}_2\text{O}_3)$  eutectics. *J. Am. Ceram. Soc.*, 1986, **69**, 252–255.
14. Dickey, E. C., Frazer, C. S., Watkins, T. R. and Hubbard, C. R., Residual stresses in high temperature ceramic eutectics. *J. Eur. Ceram. Soc.*, 1999, **19**(13–14), 2503–2509.
15. Messerschmidt, U., Baufeld, B. and Baither, D., Plastic deformation of cubic zirconia single crystals. *Key Eng. Mater.*, 1998, **153–154**, 143–182.
16. Wolfenden, A., Measurement and analysis of elastic and anelastic properties of alumina and silicon carbide. *J. Mater. Sci.*, 1997, **32**, 2275–2282.
17. Argon, A. S., Yi, J. and Sayir, A., Creep resistance of directionally solidified ceramic eutectics of  $\text{Al}_2\text{O}_3/c\text{-ZrO}_2$  with sub-micron columnar morphologies. *Mater. Sci. Eng.*, 2001, **A319–321**, 838–842.
18. Pollock, T. M. and Argon, A. S., Creep resistance of CMSX-3 nickel base superalloy single crystals. *Acta Metall.*, 1992, **40**, 1–30.
19. Nabarro, F. R. N., Steady state diffusional creep. *Phil. Mag.*, 1967, **16**, 231–238.
20. Li, J. C. M., Kinetics and dynamics in dislocation plasticity. In *Dislocation Dynamics*, ed. A. R. Rosenfield, G. T. Hahn, A. L. Bement and R. I. Jaffee Jr. McGraw-Hill, New York, 1968, pp. 87–116.
21. Bardeen, J. and Herring, C., Diffusion in alloys and the Kirkendall effect. In *Imperfections in Nearly Perfect Crystals*, ed. W. Shockley, J. H. Hollomon, R. Maurer and F. Seitz. John Wiley & Sons, NY, 1952, pp. 261–288.
22. Brown, L. M. and Stobbs, W. M., Modeling structural changes in deformed dispersion strengthened crystals. In *Constitutive Equations in Plasticity*, ed. A. S. Argon. M.I.T. Press, Cambridge, MA, 1975, pp. 387–429.
23. Friedel, J., *Dislocations*. Addison-Wesley, Reading, MA, 1964, p. 104.
24. Groves, G. W. and Kelly, A., Change of shape due to dislocation climb. *Phil. Mag.*, 1969, **19**, 977–986.
25. Oishi, Y. and Kingery, W. D., Self-diffusion of oxygen in single crystal and polycrystalline aluminum oxide. *J. Chem. Phys.*, 1960, **33**, 480–486.